RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)
B.A./B.Sc. SIXTH SEMESTER EXAMINATION, JULY 2021
THIRD YEAR [BATCH 2018-21]

Date : 13/07/2021 **MATHEMATICS**
Time : 11.00 am – 2.00 pm **Paper** : MTM P 8 Full Marks:70

Instructions to the students

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group A (Analytical Statics)

Answer any **five** questions from Q1 to Q7.

[5x6=30]

1. A system of coplanar forces has moments H, $2H$ about the points $(2a, 0), (0, a)$ respectively referred to fixed rectangular axes. The total resolved parts of the forces about the line $y = x$ vanishes. Find the points in which the line of action of the resultant meets the coordinate axes. [6]

2. A heavy uniform rod of length $2a$, rests with its ends in contact with two smooth inclined planes of inclination α and β to the horizon. If θ is the inclination of the rod to the horizon, prove by the principle of virtual work that $2\tan\theta = \cot\alpha - \cot\beta$. [6]

3. A solid of uniform density in the form of a portion of a paraboloid of revolution; bounded by a plane perpendicular to its axis is placed with its axis vertical and the vertex resting on a horizontal plane. If the paraboloid is formed by the revolution of a parabola of latus rectum $4a$, show that the equilibrium will be stable if the height of the paraboloid is less than $3a$. [6]

4. A uniform ladder of weight W, inclined to the horizon at 45^o, rests with its upper extremity against a rough vertical wall and its lower extremity on a rough ground. Prove that the least horizontal force which will move the lower end towards the wall is just greater than $\frac{W}{2}(\mu + \frac{1+\mu}{1-\mu'})$, where μ, μ' are respectively the coefficient of friction at the lower and upper ends. [6]

5. Two forces P, Q act along the straight lines whose equations are $y = x\tan\alpha,\ z = c$ and $y = -x\tan\alpha,\ z = -c$ respectively. Show that their central axis is $y = x\frac{P-Q}{P+Q}\tan\alpha,\ \frac{z}{c} = \frac{P^2 - Q^2}{P^2 + 2PQ\cos 2\alpha + Q^2}$. [6]

6. A heavy uniform string rests on the upper surface of a rough vertical circle of radius a, and partly hangs vertically. If one end be at the highest point of the circle, show that the greatest length that can hang freely is
$$\frac{2a\mu + (\mu^2 - 1)ae^{\frac{\mu\pi}{2}}}{1+\mu^2},$$
where μ is the coefficient of friction. [6]

7. A telegraph wire is made of a given material, and such a length l is stretched between two posts, distant d apart and of same height, as will produce the least possible tension at the posts. Show that $l = \frac{d}{\lambda}\sinh\lambda$, where λ is given by the equation $\lambda\tanh\lambda = 1$. [6]

Group B (Computer fundamentals and Programming in C)

Answer any **four** questions from Q8 to Q12.

[4 x 2 = 8 marks]

8. What is the role of an operating system? [2]

9. Write a short note on ASCII Codes. [2]

10. Express in DNF: $xy' + yz' + zx'$. [2]

11. "Set of all integers does not form a Boolean Algebra under addition and multiplication" – Justify! [2]

12. Draw a switching circuit that realizes the function: $f(x, y, z) = x(y + z) + zx'$. [2]

Answer any **three** questions from Q13 to Q16.

[3 x 4 = 12 marks]

13. Write a recursion function in C programming language for finding the value of the factorial of n, where n is any positive integer. [4]

14. Prove that in a Boolean algebra, the associate laws with respect to the addition and multiplication operations hold. [2+2]

15. Let B be the set of all positive divisors of 48. Define the binary operation + and . on B by $a + b = lcm(a, b)$ and $a.b = gcd(a, b)$ and unary operation $'$ by $a' = \frac{48}{a}$. Show that $(B, +, ., ')$ is not a Boolean algebra. [4]

16. Let $f(x, y, z)$ be a Boolean function such that f takes 0 if and only if at least two of the variables take non-zero value. Express f in CNF. What will be its DNF? [4]

Group C

(Answer any one of the following units)

Unit- I (Topology)

Answer all the questions. Maximum you can score is 20.

17. (a) Let $A = \{-\frac{1}{n} : n \in \mathbb{N}\}$. Find the limit points of A in $\mathbb{R}$ with lower limit topology and in $\mathbb{R}$ with upper limit topology.

    (b) Consider the topology $\tau = \{U \subseteq \mathbb{R} : U \subseteq [0, 1]\} \cup \{\mathbb{R}\}$ on $\mathbb{R}$. Characterize all dense sets in $(\mathbb{R}, \tau)$. [(2+2)+3=7]

18. (a) Suppose (X, τ) is a 2nd countable space and $A \subseteq X$ is uncountable. Prove that $\exists\, x \in A$ such that x is a limit point of A.

    (b) Give an example to show that 18(a) fails if (X, τ) is just a separable space. (Hint: Every topological space is a subspace of a separable space). [3+2=5]

19. (a) Consider the map $f : (\mathbb{R}, \tau_f) \to (\mathbb{R}, \tau)$ defined by $f(x) = |x|\ \forall x \in \mathbb{R}$ where τ_f and τ respectively denote the cofinite topology and usual topology on $\mathbb{R}$. Verify whether f is continuous, open and closed.

    (b) Suppose $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x + x^3\ \forall x \in \mathbb{R}$.
    Prove that f is a homeomorphism. [(2+2+2)+6=12]

Unit- II (Graph Theory)

Answer all questions, maximum you score 20 marks.

20. Show that the graph K_5 is non-planar. [3]

21. G is a loop-free connected graph with 4 vertices each of degree 3. How many regions this graph can be divided? [1]

22. Prove that a non-empty connected graph G is Eulerian if and only if G is the union of some edges-disjoint circuits. [3]

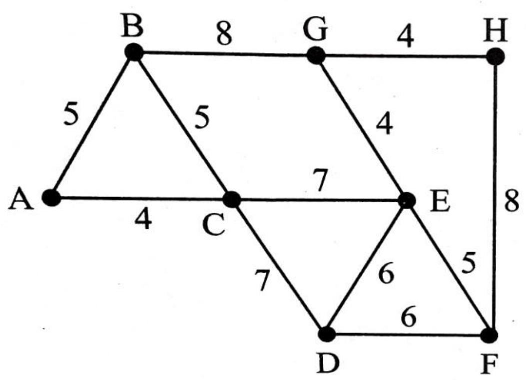
23. If a simple graph G with n vertices has more than $\frac{1}{2}(n-1)(n-2)$ edges, then prove that G is connected. [3]

24. How many vertices are there in a graph with 15 edges if each vertex is of degree 3? Justify your answer. [2]

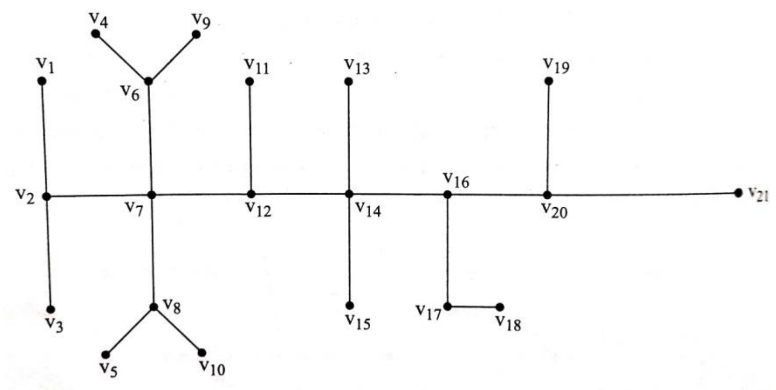
25. Draw two 3-regular graphs with eight vertices. [2]

26. Prove that every tree T with at least two vertices has at least two pendant vertices. [3]

27. Use Kruskal's algorithm to find a minimal spanning tree for the following connected weighted graph (the weight of each edge is given in terms of kilometres) : [3]



28. Find the eccentricity of all vertices, radius and centre of the following graph : [4]



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